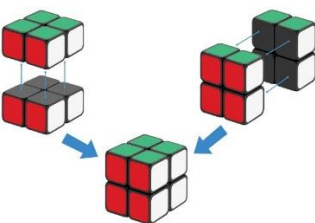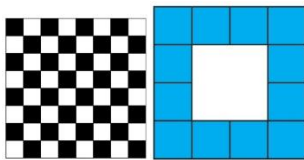| Year 5			
Year 5	**Concrete**	**Pictorial**	**Abstract**
Column addition with whole	Use place value equipment to represent additions. *Add a row of counters onto the place value grid to show 15,735 + 4,012.* 	Represent additions, using place value equipment on a place value grid alongside written methods. *I need to exchange 10 tens for a 100.*	Use column addition, including exchanges.
Representing additions		Bar models represent addition of two or more numbers in the context of problem solving. 	Use approximation to check whether answers are reasonable. *I will use 23,000 + 8,000 to check.*
Adding tenths	Link measure with addition of decimals. *Two lengths of fencing are 0·6 m and 0·2 m.* *How long are they when added together?* 0·6 m 0·2 m 	Use a bar model with a number line to add $0·6 + 0·2 = 0·8$ *6 tenths + 2 tenths = 8 tenths*	Understand the link with adding fractions. $\dfrac{6}{10} + \dfrac{2}{10} = \dfrac{8}{10}$ *6 tenths + 2 tenths = 8 tenths* $0·6 + 0·2 = 0·8$

Adding decimals using column addition	Use place value equipment to represent additions. *Show 0·23 + 0·45 using place value counters.*	Use place value equipment on a place value grid to represent additions. Represent exchange where necessary. Include examples where the numbers of decimal places are different. 	Add using a column method, ensuring that children understand the link with place value. Include exchange where required, alongside an understanding of place value. Include additions where the numbers of decimal places are different. 3.4 + 0.65 = ?
Column subtraction with whole numbers	Use place value equipment to understand where exchanges are required. *2,250 – 1,070* 	Use place value equipment on a grid alongside the calculation, including exchanges where required. *15,735 − 2,582 = 13,153* Now subtract the 10s. Exchange 1 hundred for 10 tens. Subtract the 100s, 1,000s and 10,000s. 	Use column subtraction methods with exchange where required. *62,097 − 18,534 = 43,563*
Checking strategies and		Bar models represent subtractions in problem contexts, including 'find the difference'. 	Children can explain the mistake made when the columns have not been ordered correctly. Use approximation to check calculations. *I calculated 18,000 + 4,000 mentally to check my subtraction.*

Choosing efficient methods			To subtract two large numbers that are close, children find the difference by counting on. *2,002 − 1,995 = ?* Use addition to check subtractions. *I calculated 7,546 − 2,355 = 5,191.*
Subtracting decimals	Explore complements to a whole number by working in the context of length. I m − ☐ m = ☐ m *1 − 0·49 = ?*	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 − 2·25 = ?* 	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 − 3·75 = ?* O · Tth Hth Thth 3 · 9 2 1 − 3 · 7 5 0 ·
Understanding factors	Use cubes or counters to explore the meaning of 'square numbers'. *25 is a square number because it is made from 5 rows of 5.* Use cubes to explore cube numbers. 8 is a cube number.	Use images to explore examples and non-examples of square numbers. *8 × 8 = 64* *$8^2 = 64$* *12 is not a square number, because you cannot multiply a whole number by itself to make 12.*	Understand the pattern of square numbers in the multiplication tables. Use a multiplication grid to circle each square number. Can children spot a pattern?

Multiplying by 10, 100 and 1,000	Use place value equipment to multiply by 10, 100 and 1,000 by unitising. 4 × 1 = 4 ones = 4 4 × 10 = 4 tens = 40 4 × 100 = 4 hundreds = 400	Understand the effect of repeated multiplication by 10.	Understand how exchange relates to the digits when multiplying by 10, 100 1,000. | H	T	O
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	1	7	 *17 × 10 = 170* *17 × 100 = 17 × 10 × 10 = 1,700* *17 × 1,000 = 17 × 10 × 10 × 10 = 17,000*
Multiplying by multiples of 10, 100 and 1,000	Use place value equipment to explore multiplying by unitising. *5 groups of 3 ones is 15 ones.* *5 groups of 3 tens is 15 tens.* *So, I know that 5 groups of 3 thousands would be 15 thousands.*	Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000. *4 × 3 = 12* *6 × 4 = 24* *4 × 300 = 1,200* *6 × 400 = 2,400*	Use known facts and unitising to multiply. *5 × 4 = 20* *5 × 40 = 200* *5 × 400 = 2,000* *5 × 4,000 − 20,000* *5,000 × 4 = 20,000*
Multiplying up to 4-digit numbers by a single digit	Explore how to use partitioning to multiply efficiently. *8 × 17 = ?* 8 × 10 = 80 8 × 7 = 56 80 + 56 = 136 *So, 8 × 17 = 136*	Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s. 	H	T	O
---|---|---|	Use an area model and then add the parts. |	100	60	3

5	100 × 5 = 500	60 × 5 = 300	3 × 5 = 15 |
			
			Use a column multiplication, including any required exchanges.
			
			  1 3 6
			× 6
			  8 1 6
			    2 3

Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. *23 × 15 = ?* 10 × 15 = 150 10 × 15 = 150 3 × 15 = 45 There are 345 bottles of milk in total. H T O 1 5 0 1 5 0 + 4 5 3 4 5 *23 × 15 = 345*	Use an area model and add the parts. *28 × 15 = ?* 		20 m	8 m
---	---	---
10 m	20 × 10 = 200 m²	8 × 10 = 80 m²
5 m	20 × 5 = 100 m²	8 × 5 = 40 m² | H T O 2 0 0 1 0 0 8 0 + 4 0 4 2 0 *28 × 15 = 420*	Use column multiplication, ensuring understanding of place value at each stage. 3 4 × 2 7 2 3₂8 34 × 7 3 4 × 2 7 2 3₂8 34 × 7 6 8 0 34 × 20 3 4 × 2 7 2 3₂8 34 × 7 6 8 0 34 × 20 9 1 8 34 × 27
Multiplying up to 4-digits by 2-digits		Use the area model then add the parts. |	100	40	3
10			
2				 143 × 12 = 1,716 There are 1,716 boxes of cereal in total. Th H T O 1 0 0 0 4 0 0 2 0 0 8 0 3 0 + 6 1 7 1 6 *143 × 12 = 1,716*	Use column multiplication, ensuring understanding of place value at each stage. 1 4 3 × 1 2 2 8 6 143 × 2 1 4 3 0 143 × 10 1 7 1 6 143 × 12 Progress to include examples that require multiple exchanges as understanding, confidence and fluency build. *1,274 × 32 = ?* *First multiply 1,274 by 2.* 1 2 7 4 × 3 2 2 5₁4 8 1,274 × 2 1 2 7 4 × 3 2 2 5₁4 8 1,274 × 2 3 8₂2₁2 0 1,274 × 30 *Then multiply 1,274 by 30.* *Finally, find the total.* 1 2 7 4 × 3 2 2 5₁4 8 1,274 × 2 3 8₂2₁2 0 1,274 × 30 4 0 7 6 8 1,274 × 32 *1,274 × 32 = 40,768*
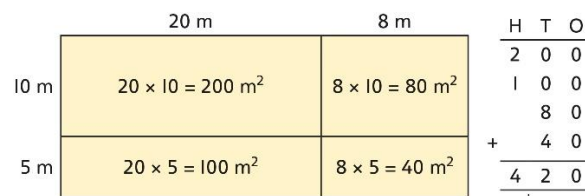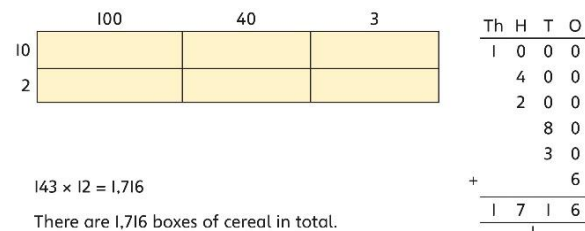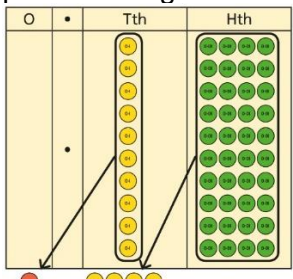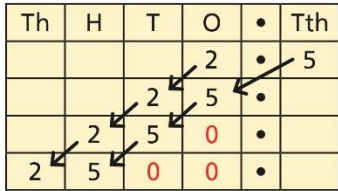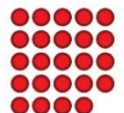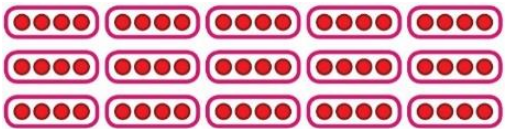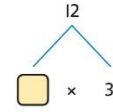
Multiplying decimals by 10, 100 and 1,000	Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.	Represent multiplication by 10 as exchange on a place value grid. $0·14 \times 10 = 1·4$	Understand how this exchange is represented on a place value chart. $2·5 \times 10 = 25$ $2·5 \times 100 = 250$ $2·5 \times 1,000 = 2,500$
Understanding factors and prime numbers	Use equipment to explore the factors of a given number. $24 \div 3 = 8$ $24 \div 8 = 3$ 8 and 3 are factors of 24 because they divide 24 exactly. $24 \div 5 = 4$ remainder 4. 5 is not a factor of 24 because there is a remainder.	Understand that prime numbers are numbers with exactly two factors. $13 \div 1 = 13$ $13 \div 2 = 6 \ r \ 1$ $13 \div 4 = 4 \ r \ 1$ 1 and 13 are the only factors of 13. 13 is a prime number.	Understand how to recognise prime and composite numbers. *I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.* *I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.* *I know that 1 is not a prime number, as it has only 1 factor.*
inverse operations link multiplication,	Use equipment to group and share and to explore the calculations that are present. *I have 28 counters.* *I made 7 groups of 4. There are 28 in total.* *I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.* *I have 28 in total. I made groups of 4. There are 7 equal groups.*	Represent multiplicative relationships and explore the families of division facts. $60 \div 4 = 15$ $60 \div 15 = 4$	Represent the different multiplicative relationships to solve problems requiring inverse operations. $22 \div ? = 2$ $22 \div 2 = ?$ $? \div 2 = 22$ $? \div 22 = 2$

Dividing whole numbers by 10, 100 and 1,000	Use place value equipment to support unitising for division. *4,000 ÷ 1,000* 4,000 1,000 × ▢ *4,000 is 4 thousands.* *4 × 1,000= 4,000* *So, 4,000 ÷ 1,000 = 4*	Use a bar model to support dividing by unitising. *380 ÷ 10 = 38* 380 ? ? ? ? ? ? ? ? ? ? 380 10 × ▢ *380 is 38 tens.* *38 × 10 = 380* *10 × 38 = 380* *So, 380 ÷ 10 = 38*	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000. | Th	H	T	O
3	2	0	0 | *3,200 ÷ 100 = ?* *3,200 is 3 thousands and 2 hundreds.* *200 ÷ 100 = 2* *3,000 ÷ 100 = 30* *3,200 ÷ 100 = 32* *So, the digits will move two places to the right.*
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising. *15 ones put into groups of 3 ones. There are 5 groups.* *15 ÷ 3 = 5* *15 tens put into groups of 3 tens. There are 5 groups.* *150 ÷ 30 = 5*	Represent related facts with place value equipment when dividing by unitising. *180 is 18 tens.* *18 tens divided into groups of 3 tens. There are 6 groups.* *180 ÷ 30 = 6* *12 ones divided into groups of 4. There are 3 groups.* *12 hundreds divided into groups of 4 hundreds. There are 3 groups.* *1200 ÷ 400 = 3*	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. *3,000 ÷ 5 = 600* *3,000 ÷ 50 = 60* *3,000 ÷ 500 = 6* *5 × 600 = 3,000* *50 × 60 = 3,000* *500 × 6 = 3,000*
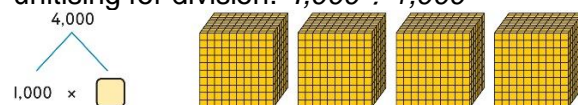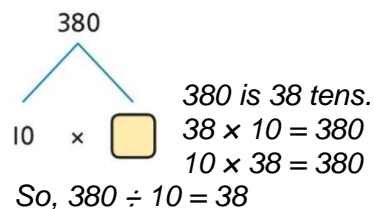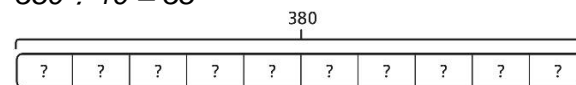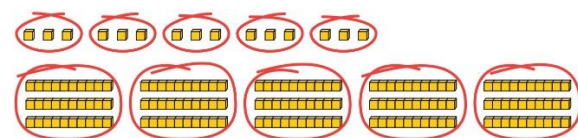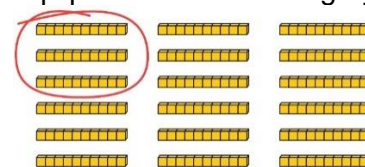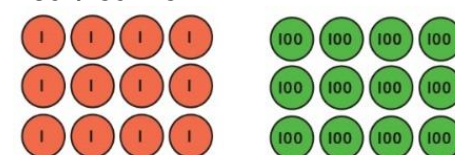
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. *268 ÷ 2 = ?* *There is 1 group of 2 hundreds.* *There are 3 groups of 2 tens.* *There are 4 groups of 2 ones.* *264 ÷ 2 = 134*	Use place value equipment on a place value grid alongside short division. The model uses grouping. Lay out the problem as a short division. *There is 1 group of 4 in 4 tens.* *There are 2 groups of 4 in 8 ones.* 	Use short division for up to 4-digit numbers divided by a single digit. $$0\ 5\ 5\ 6$$ $$7\ \overline{)\ 3\ {}^{3}8\ {}^{3}9\ {}^{4}2}$$ *3,892 ÷ 7 = 556* Use multiplication to check. *556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892*
Understanding remainders	Understand remainders using concrete versions of a problem. *80 cakes divided into trays of 6.* *80 cakes in total. They make 13 groups of 6, with 2 remaining.*	remainders as the last remaining 1s. 	In problem solving contexts, represent divisions including remainders with a bar model. *683 = 136 × 5 + 3* *683 ÷ 5 = 136 r 3*
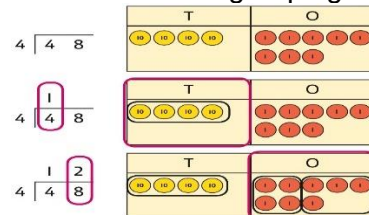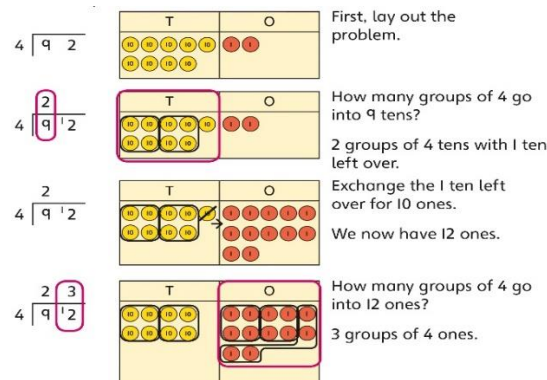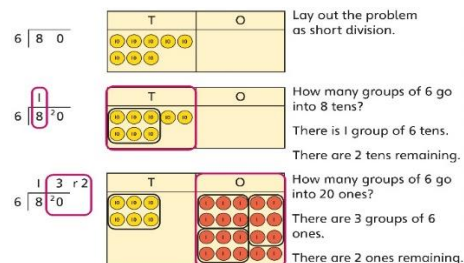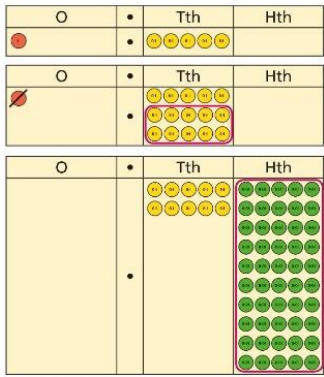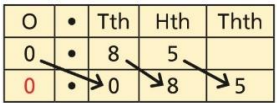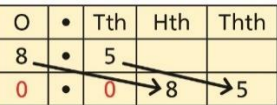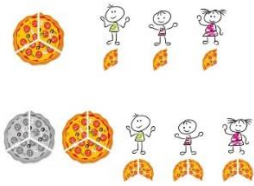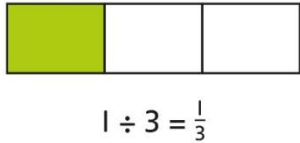
	Concrete	Pictorial	Abstract
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. *2 ones are 20 tenths.* *20 tenths divided by 10 is 2 tenths.*	Represent division using exchange on a place value grid. *1·5 is 1 one and 5 tenths.* *This is equivalent to 10 tenths and 50 hundredths.* *10 tenths divided by 10 is 1 tenth.* *50 hundredths divided by 10 is 5 hundredths.* *1·5 divided by 10 is 1 tenth and 5 hundredths.* $1·5 \div 10 = 0.15$	Understand the movement of digits on a place value grid. $0·85 \div 10 = 0·085$ $8·5 \div 100 = 0·085$
Understanding the relationship between	Use sharing to explore the link between fractions and division. *1 whole shared between 3 people.* *Each person receives one-third.* 	Use a bar model and other fraction representations to show the link between fractions and division. $1 \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$
Year 6			
Year 6	**Concrete**	**Pictorial**	**Abstract**

Comparing and selecting efficient methods	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.
	
	| M
		HTh	TTh	Th	H	T	O |
							●● ●●●● ● ● ●●● ●
								Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation.
								Compare written and mental methods alongside place value representations
								
								+1 hour
								+8 minutes
								12:05 13:05 13:13
								
								+ 3,000 + 500 + 20 + 2
								40,265 43,265
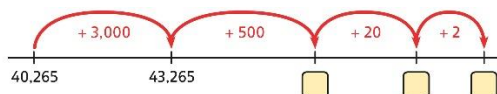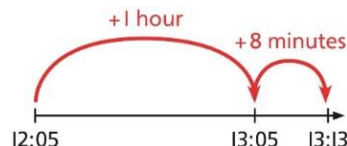								
								Use bar model and number line representations to model addition in problem-solving and measure contexts.
									Use column addition where mental methods are not efficient. Recognise common errors with column addition.
									
									32,145 + 4,302 = ?
									
									TTh Th H T O TTh Th H T O
									  3 2 1 4 5 3 2 1 4 5
									+ 4 3 0 2 + 4 3 0 2
									  3 6 4 4 7 7 5 1 6 5
									
									Which method has been completed accurately? What mistake has been made?
									
									Column methods are also used for decimal additions where mental methods are not efficient.
									
									H T O · Tth Hth
									1 4 0 · 0 9
									+ 4 9 · 8 9
									1 8 9 · 9 8

Selecting mental methods for larger numbers where appropriate	Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods. | M	HTh	TTh	Th	H	T	O | ●● ●●●● ● ● ●●● ● *2,411,301 + 500,000 = ?* *This would be 5 more counters in the HTh place.* *So, the total is 2,911,301.* *2,411,301 + 500,000 = 2,911,301*	Use a bar model to support thinking in addition problems. *257,000 + 99,000 = ?* ? £257,000	£100,000 *I added 100 thousands then subtracted 1 thousand.* *257 thousands + 100 thousands = 357 thousands* *257,000 + 100,000 = 357,000* *357,000 – 1,000 = 356,000* *So, 257,000 + 99,000 = 356,000*	Use place value and unitising to support mental calculations with larger numbers. *195,000 + 6,000 = ?* *195 + 5 + 1 = 201* *195 thousands + 6 thousands = 201 thousands* *So, 195,000 + 6,000 = 201,000*
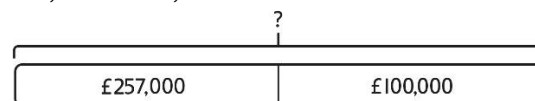
Understanding order of operations in calculations	Use equipment to model different interpretations of a calculation with more than one operation. Explore different results. *3 × 5 − 2 = ?* 3 × 5 − 2 ↓ ↓ 3 × 3 = 9 3 × 5 − 2 ↓ ↓ 15 − 2 = 13	Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations. This can be written as: 16 × 4 + 16 × 6 16 × 4 + 16 × 6 64 + 96 = 160	Understand the correct order of operations in calculations without brackets. Understand how brackets affect the order of operations in a calculation. *4 + 6 × 16* *4 + 96 = 100* *(4 + 6) × 16* *10 × 16 = 160*
Comparing and selecting efficient methods	Use counters on a place value grid to represent subtractions of larger numbers. 	Compare subtraction methods alongside place value representations. Use a bar model to represent calculations, including 'find the difference' with two bars as comparison. 	Compare and select methods. Use column subtraction when mental methods are not efficient. Use two different methods for one calculation as a checking strategy. Use column subtraction for decimal problems, including in the context of measure.
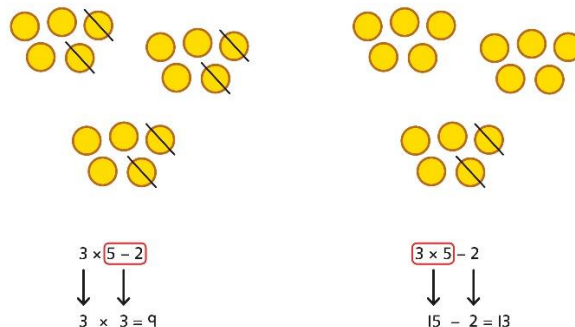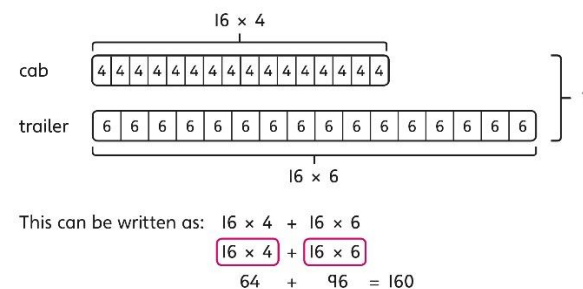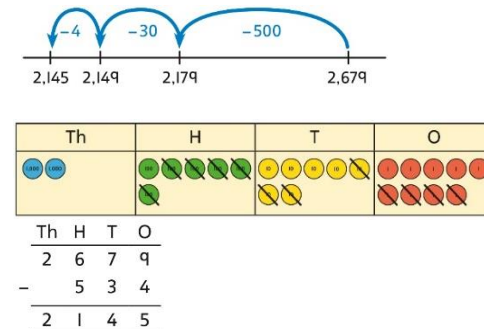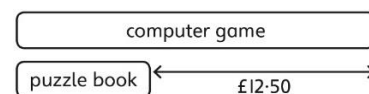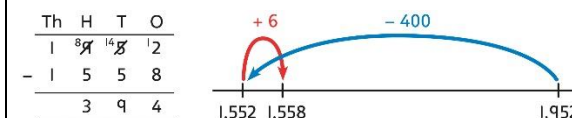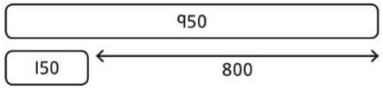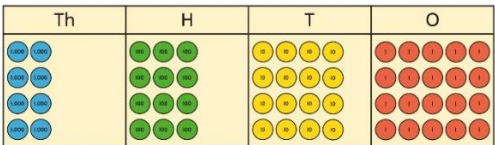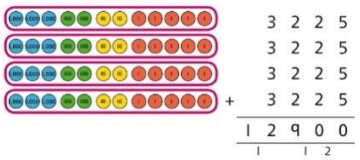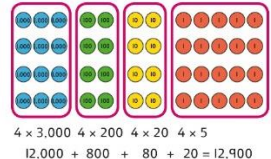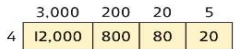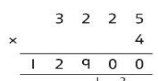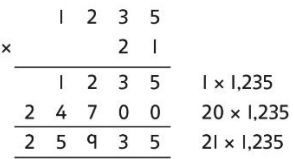
Subtracting mentally with larger numbers		Use a bar model to show how unitising can support mental calculations. *950,000 − 150,000* *That is 950 thousands − 150 thousands* *So, the difference is 800 thousands.* *950,000 − 150,000 = 800,000*	Subtract efficiently from powers of 10. *10,000 − 500 = ?*
Multiplying up to a 4-digit number by a single digit	Use equipment to explore multiplications. *4 groups of 2,345* *This is a multiplication:* *4 × 2,345* *2,345 × 4*	Use place value equipment to compare methods. 	Understand area model and short multiplication. Compare and select appropriate methods for specific multiplications.
Multiplying up to a 4-digit number by a 2-digit number		Use an area model alongside written multiplication. 	Use compact column multiplication with understanding of place value at all stages.

Using knowledge of factors and partitions to compare methods for	Use equipment to understand square numbers and cube numbers. $5 \times 5 = 5^2 = 25$ $5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$	Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately. Represent and compare methods using a bar model.	Use a known fact to generate families of related facts. Use factors to calculate efficiently. 15×16 $= 3 \times 5 \times 2 \times 8$ $= 3 \times 8 \times 2 \times 5$ $= 24 \times 10$ $= 240$
Multiplying by 10, 100 and 1,000	Use place value equipment to explore exchange in decimal multiplication. $0.3 \times 10 = ?$ 0.3 is 3 tenths. 10×3 tenths are 30 tenths. 30 tenths are equivalent to 3 ones.	Understand how the exchange affects decimal numbers on a place value grid. $0.3 \times 10 = 3$	Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000. $8 \times 100 = 800$ $8 \times 300 = 800 \times 3$ $\qquad = 2,400$ $2.5 \times 10 = 25$ $2.5 \times 20 = 2.5 \times 10 \times 2$ $\qquad = 50$
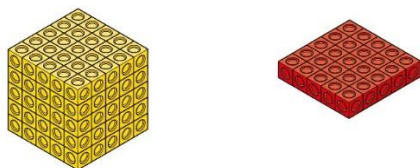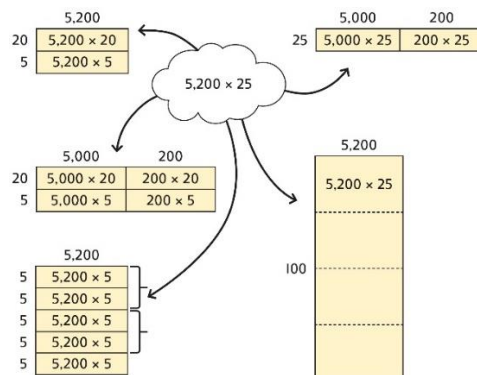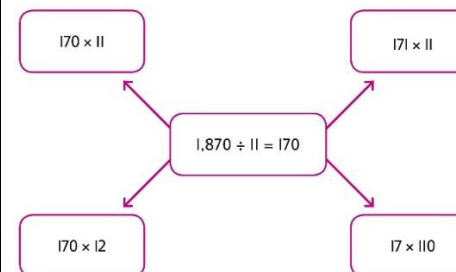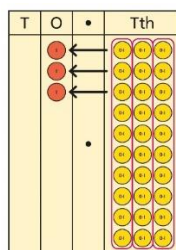
Multiplying decimals

Explore decimal multiplications using place value equipment and in the context of measures.

3 groups of 4 tenths is 12 tenths.
4 groups of 3 tenths is 12 tenths.

1·3 cm 1·3 cm 1·3 cm 1·3 cm
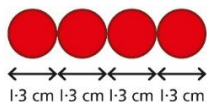
4 × 1 cm = 4 cm
4 × 0·3 cm = 1.2 cm
4 × 1·3 = 4 + 1·2 = 5·2 cm

Represent calculations on a place value grid.

$3 \times 3 = 9$

$3 \times 0\cdot3 = 0\cdot9$

Understand the link between multiplying decimals and repeated addition.
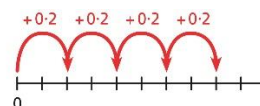
Use known facts to multiply decimals.
4 × 3 = 12
4 × 0·3 = 1·2
4 × 0·03 = 0·12
I know that 18 × 4 = 72.
This can help me work out:

1·8 × 4 = ?
18 × 0·4 = ?
180 × 0·4 = ?
18 × 0·04 = ?

Use a place value grid to understand the effects of multiplying decimals.

Understanding factors

Use equipment to explore different factors of a number.

4 is a factor of 24 but is not a factor of 30.

Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.

17 ÷ 2 = 8 r 1   17 ÷ 3 = 5 r 2   17 ÷ 4 = 4 r 1   17 ÷ 5 = 3 r 2
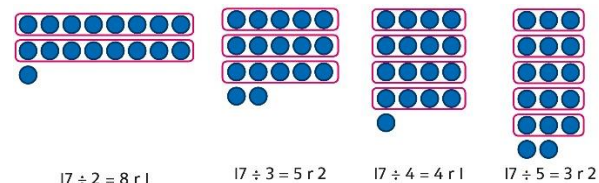
Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.

Dividing by a single digit

Use equipment to make groups from a total.
There are 78 in total.
There are 6 groups of 13.
There are 13 groups of 6.

6 × ? = 132

132 = 120 + 12

132 ÷ 6 = 20 + 2 = 22
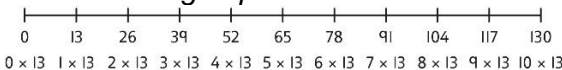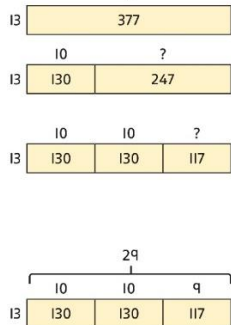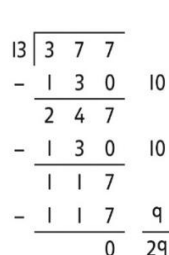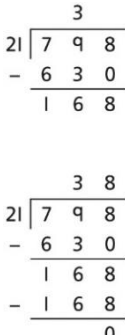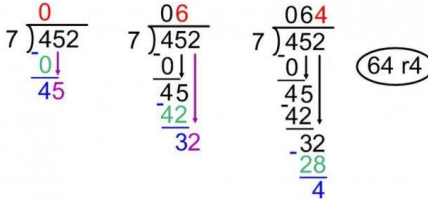
Use short division to divide by a single digit.

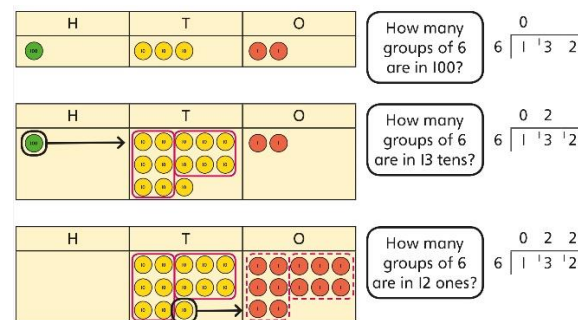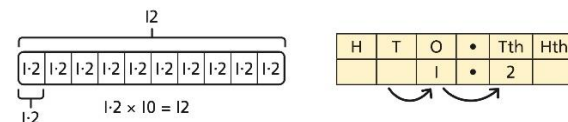|			
Dividing by a 2-digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. *1,260 ÷ 14 = ?* 1,260 *1,260 ÷ 2 = 630* *630 ÷ 7 = 90* *1,260 ÷ 14 = 90*	Use factors and repeated division where appropriate. *2,100 ÷ 12 = ?* 2,100 → ÷2 → ÷6 → 2,100 → ÷6 → ÷2 → 2,100 → ÷3 → ÷4 → 2,100 → ÷4 → ÷3 → 2,100 → ÷3 → ÷2 → ÷2 →

Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.
	
	182 divided into groups of 13.
	There are 14 groups.
	
	0 13 26 39 52 65 78 91 104 117 130
	0×13 1×13 2×13 3×13 4×13 5×13 6×13 7×13 8×13 9×13 10×13
		Use an area model alongside written division to model the process.
		
		377 ÷ 13 = ?
		
		13 | 377
		
		10 ?
		13 | 130 247
		
		10 10 ?
		13 | 130 130 117
		
		29
		10 10 9
		13 | 130 130 117
		
		377 ÷ 13 = 29
			Use long division where factors are not useful (for example, when dividing by a 2-digit prime number).
			377 ÷ 13
			
			$$13 \overline{)377}$$
			− 1 3 0 10
			2 4 7
			− 1 3 0 10
			1 1 7
			− 1 1 7 9
			0 29
			
			3
			21 | 7 9 8
			− 6 3 0
			1 6 8
			
			3 8
			21 | 7 9 8
			− 6 3 0
			1 6 8
			− 1 6 8
			0
			
			377 ÷ 13 = 29
			A slightly different layout may be used, with the division completed above rather than at the side.
			
			0 06 064
			7)452 7)452 7)452 (64 r4)
			0↓ 0↓ 0↓
			45 45 45
			    42↓ 42↓
			    32 -32
			        28
			        4
			
			Step 1: "How many times?"
			Step 2: "Multiply"
			Step 3: "Subtract"
			Step 4: "Drop it down"
			(repeat steps for each number, left to right)
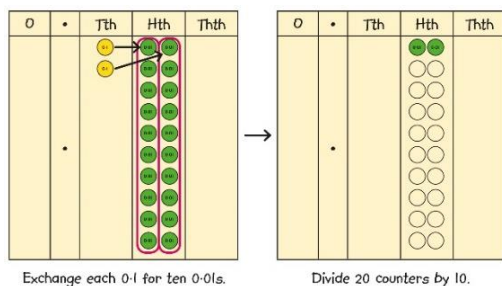
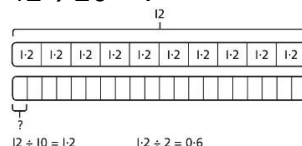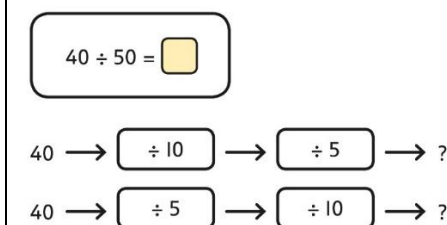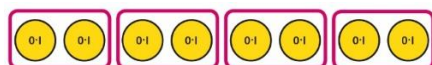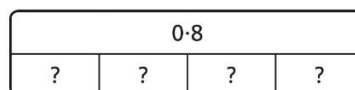|			
Dividing by 10, 100 and 1,000	Use place value equipment to explore division as exchange. Exchange each 0·1 for ten 0·01s. Divide 20 counters by 10. *0·2 is 2 tenths.* *2 tenths is equivalent to 20 hundredths.* *20 hundredths divided by 10 is 2 hundredths.*	Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid. $1·2 \times 10 = 12$ Understand how to divide using division by 10, 100 and 1,000. *$12 \div 20 = ?$* $12 \div 10 = 1·2$ $1·2 \div 2 = 0·6$	Use knowledge of factors to divide by multiples of 10, 100 and 1,000. $40 \div 50 = \square$ $40 \rightarrow \div 10 \rightarrow \div 5 \rightarrow ?$ $40 \rightarrow \div 5 \rightarrow \div 10 \rightarrow ?$ *$40 \div 5 = 8$* *$8 \div 10 = 0·8$* *So, $40 \div 50 = 0·8$*
Dividing decimals	Use place value equipment to explore division of decimals. *8 tenths divided into 4 groups. 2 tenths in each group.*	Use a bar model to represent divisions. $4 \times 2 = 8$ $8 \div 4 = 2$ So, $4 \times 0·2 = 0·8$ $0·8 \div 4 = 0·2$	Use short division to divide decimals with up to 2 decimal places.